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Option-Implied Adjusted Volatility Using Modified Generalised Leland Models: An Empirical Study on Dow Jones Industrial Average Index Options

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ABSTRACT

This study investigates the relative option pricing performance of Modified Generalised Leland models. We employ non-parametric mechanism within the conventional option-pricing framework based on the Leland models to assure realistic pricing of options. This study extends the models by developing Modified Generalised Leland models based on the implied adjusted volatility introduced in Leland models. The proposed models are developed to incorporate the transaction costs rate in the integrated model-free framework. Relevant sample data extracted from the Dow Jones Industrial Average index options is tested in this study. We find that the option-implied adjusted volatility, which is priced using the Modified Generalised Leland models, delivers a significant improvement to the option valuation accuracy.

Keywords: Model-free,modelling, options, transaction costs, volatility.

1. Introduction

The vast practicality of option pricing is unquestionable. This has in fact been the focus of many researchers as well as market practitioners. The development of option pricing model has been phenomenal especially after the influential study done in 1973. The model introduced by Black and Scholes (1973) and Merton (1973), i.e. the Black-Scholes-Merton (BSM) model has been approved as an established paragon in finance. BSM is the most extensively used model, despite its unrealistic and astringent assumptions. A study conducted by Galai (1983) showed that the BSM model produced ample pricing bias systematically. This has sparked a plethora of study to improve the option pricing model. The generalisation of the BSM model leads to the sporadic growth of evolution in the modern parametric option pricing models.

The modern parametric option pricing models which attempt to generalise and relax the assumptions built within the BSM model has demonstrated to be comprehensive instrument in pricing options. However, the generalisations often lead to complexly overfitted and misspecified class of parametric models. Lajbcygier (1999) highlighted that the modern parametric approaches tend to be too complex. They fail to outperform even simple, easy models. These generalised models which utilise unrealistic parameters are exposed to over-parametrisation problem. This is understandable since they incline to produce parameters inconsistent with underlying time series without costing the elimination of the systematic pricing bias. The justification is provided in Radzikowski (2000). Jankova (2018) also highlighted the constant volatility assumption problem. Besides, Bates (2000) reported that the risk of over-fitting the option data is further pronounced by the use of over-parameterised models.

The quest to find one ideal and eloquent option pricing model to explain option prices seems to be impossible at this pace. This has urged many researches to consider the complementary non-parametric approach instead. This approach presumes no complex model in deducing prices, unlike the conventional parametric approach. It is apparent that the intricate parametrisation feature of the parametric approach serves the main door to over-parametrisation problem. Alternatively, the option price is directly deduced from the historical data based on the non-parametric approach. Rational and realistic option pricing, in spite of that, is not assured in non-parametric method. It is reported in Chen and Xu (2014) that the nonparametric model did not remove pricing error effect. Hence, Radzikowski (2000) underlined that the ultimate option pricing model is not at one of the ends, but may be at the middle which integrates both approaches. This research attempts to differ from others. Instead of focusing on how to deliberately improve the existing work expansion on option pricing model in the parametric model framework, this study endeavors to develop a model that employs non-parametric mechanism while still conforming to the conventional option pricing framework to assure realistic pricing of options.

The model introduced by Black and Scholes (1973) and Merton (1973) rests on the assumptions of no arbitrage, pricing log-normality and frictionless trading. Therefore, the introduction of this Black-Scholes model in the 1970s invited many criticisms. Owing to the pitfalls of the BSM model in pricing options, the volatility implied from this model is unable to directly proxy the true expectation of future realised volatility, see Shu and Zhang (2003). Henceforth, a number of models have been developed to modify and cater the pitfall imbued within the BSM model. Leland (1985) is among the first that improved the BSM model by developing a hedging strategy that incorporates an adjusted volatility. The volatility is adjusted with respect to the length of rebalance intervals, proportional transaction cost rate and the volatility of the underlying asset. One of the BSM assumptions is zero transaction costs. Leland (1985) model relaxed the assumption by forcing the length of rebalance intervals to approach zero, so that zero hedging error can be achieved in the limit. Even though the idea is quite relevant, this model does not integrate the initial cost of trading into the assumptions. In response to the drawbacks of the original model, Leland (2007) provided two adjustments; namely Leland cash model and Leland stock model. In these models, he explicitly considered initial costs of trading into the assumptions that the initial portfolio is either consists of all cash or all stock position.

Despite the fact that BSM model has a major flaw, yet it is still acknowledged by many studies as a relevant option pricing model. Many other models have been developed as extensions of the BSM model. Many empirical studies use BSM model as a benchmark or as an interpolation tool in investigating the performance of various option pricing models. See Christoffersen and Jacobs (2004), Figlewski (2002) and An and Suo (2009). The introduction of BSM model to the financial world has inspired numerous literature to examine the forecasting ability of implied volatility in time series framework, which was pointed out by Chernov (2001). This framework is shared by the Leland models. On top of that, a model which considers realistic transaction costs seems to be more suitable in handling options. It is hypothesised that using the Leland option pricing models which have both the almost identical framework to that of the BSM model and incorporate the stochastic nature of volatility in its model appears to be relevant in this study. This research employs the Leland

(1985) model and its two variations in estimating the option-implied moments, namely option-implied adjusted moments.

Existing option pricing models are extended in this study by incorporating a semiparametric framework into the Leland models. This model-guided nonparametric framework will be referred to as Modified Generalised Leland (MGL) models throughout this study. Based on MGL, this study generates new option-implied adjusted moments. Aside from that, using the three Leland models, this study attempts to combine the Leland models into a model-free framework, developed by Bakshi et al. (2003). The proposed resulting combination models are considered in the framework to reduce the model mis-specification errors introduced by the Leland models, while still deliver realistic pricing. According to Kempf et al. (2014), there is a gap in studying the hybrid portfolio made of both fully-implied and option-implied information. Thus, this study endeavours in fulfilling this gap.

Rather than utilising the BSM model directly in this study, a model-free framework based on the BSM model, which was proposed by Bakshi et al. (2003), is viewed to be applicable to be employed instead. The benchmark model is denoted as Model-Free Bakshi-Kapadia-Madan (MFBKM) throughout this study. To employ the BSM model as the benchmark model is not suitable since the main interest of this study is on option-implied information. The MFBKM model deals with both call and put option prices simultaneously, unlike the BSM model. This provides a shorter time computation-wise as well as decision-wise. For that reason, it is relevant for this study to employ the MFBKM as the comparison model throughout this empirical investigation.

The data for this study will be taken from the Dow Jones Industrial Index (DJIA). Having mainly benefited by the abundance of natural resources, highly-diversified workforce and well-developed infrastructures, US is considered as the largest manufacturer globally. Hence, considering US as the most vital world economic generator, it would be an interesting study to look forward into. This study intends to empirically investigate the index options data, specifically those that able to directly proxy the global index options market. For a better comprehension, the rest of the research is drawn into a number of sections. A brief introduction to this study is offered in the first section. Section 2 explains the data utilised in this paper. In Section 3, we illustrated the methodology used in assessing the relative performance of MGL. The main findings of this study are documented in Section 4. Last but not least, conclusion is given in Section 5.

2. Data

This paper utilises options on the Dow Jones Industrial Index (DJIA) traded daily on the Chicago Board Options Exchange (CBOE). The investigation includes all call and put options traded from January 2009 until December 2015. The underlying price used in this study will utilise the closing price of the DJIA index, whereas the actual option price is taken from the closing price of the option price. The DJIA index options track 30-blue chipped-companies index and equity options within the US economy. It represents the most heavily traded and listed in the US. Today, CBOE has become the largest options exchange in the US. Hence, it has been acknowledged as the largest options market in the world as well. Owing to that fact, this data is believed to best reflect the generic behaviour of the US option market. This fact offers us a broad-spectrum idea on the world index options market.

3. Methodology

This study focuses on examining the model-free implied volatility (MFIV), in relation to the first contract defined in MFBKM, i.e. the variance contract. The option-implied information are obtained by utilising three approaches, i.e. the Basic (trapezoidal-rule) approach, the Adapted (single-combined) approach and the Advanced (single-combined, cubic-spline) approach, programmed using MATLAB. In order to assess the model relative performance, this study considers the different Leland models, namely the Leland (1985), Leland All-Cash and Leland All-Stock models. Using the three Leland models, this study attempts to hybrid the models into model-free framework, developed by Bakshi et al. (2003). This research investigates the performance of MGL models in comparison with MFBKM as the benchmark model. In particular, the option pricing performance of MGL models, which composed of both Generalised Leland-Infused (GLI) model and the model-free implied Leland (MFIL), are investigated in this part of study. The option adjusted information implied from the MGL models is obtained and compared against those generated by the MFBKM model.

The proposed models are developed to incorporate the transaction costs rate in the hybrid model-free framework. Both of the MGL models are derived by integrating the Leland models as well as the model-free defined in Bakshi et al. (2003). The MGL model only considers the inclusion of the transaction costs rate and the rebalancing interval in the model. However, the GLI model did not explicitly consider the initial cost of trading. The MFIL model accounts for the initial cost of trading, on top of the transaction costs rate and the rebalancing interval factors. The new MFIL models incorporate the initial costs of trading into the assumptions that the initial portfolio consists of all cash and all stock positions. In this study, we denote these two models as MFIL All-Cash (LCASH) model and MFIL All-Stock (LSTOCK) model, in addition to the MFIL model which is based on Leland (1985) model, i.e. MFIL 1985 (L85).

In obtaining the option-implied adjusted volatility values using the MFIL model, cross-section of the option prices of both call and put options are first retrieved from the Leland models. The option prices are beforehand regenerated out of the wavelet transform discussed in previous section. The option-implied adjusted volatilities are then keyed in into the hybrid model. In contrast, the option prices of both call and put options generated by the wavelet transform are first used in obtaining option-implied volatilities in GLI model. The new option-implied adjusted volatilities are produced by infusing the Leland adjusted function. A new function of modified generalised model-free implied volatility is developed in this research based on the design.

3.1 The Modified Generalised Leland Function

The new MGL model is derived from the fact that the model-free option-implied volatility is the square-root of the Bakshi et al. (2003) variance contract. In Bakshi et al. (2003), they defined the variance contract as

$$VAR(t,\tau) \equiv E^q \left\{ \left(R_{t,\tau} - E^q\left[R_{t,\tau}\right]\right)^2 \right\}; \tag{1}$$

$$VAR(t,\tau) = e^{r\tau} V(t,\tau) - \mu(t,\tau)^2. \tag{2}$$

By equating the variance contract with the square of the adjusted volatility introduced in Leland (1985), we obtain

$$e^{r\tau} V(t,\tau) - \mu(t,\tau)^2 = 1 + \frac{k\sqrt{\frac{2}{\pi}}}{\sigma\sqrt{\delta t}}. \tag{3}$$

A quadratic equation can be created out of the above equation.

$$\sigma^2 + \sqrt[4]{\frac{2}{\pi}}\sigma \cdot k - e^{r\tau}V(t,\tau) + \mu(t,\tau)^2 = 0. \tag{4}$$

Based on the equation, we propose new modified generalised (MG) function to be:

$$MG = \frac{-k}{\sqrt{2\pi \cdot \Delta t}} + \sqrt{\frac{k^2}{\pi \cdot \Delta t} - 2\left(\mu^2 - e^{r\tau} \cdot V\right)} \tag{5}$$

where V represents the variance contract, k is the round-trip transaction cost rate per unit dollar of transaction and Δt is the time between hedging adjustment, i.e. the rebalancing interval. The MGL implied volatility is adjusted as above to account for several extra parameters which are not considered in the original BKM model, i.e. the transaction cost rate and the time between hedging adjustment. Particularly this model adopts the transaction cost function introduced by Leland models.

4. Results and Discussions

In this section, the relative option pricing performance of each model is assessed. The Leland All-Cash model of MFIL, which is estimated using the advanced approach, records the smallest RMSE of 15.13 per cent, 15.81 per cent and 16.06 per cent, with respect to the daily, weekly and fortnightly rebalancing. The GLI models show superior pricing performance across all approaches, compared to the Model-Free model. In contrast, the MFIL generated from the Leland All-Stock option pricing model outperforms other models when the estimation is done using the adapted approach. As a matter of fact, the MGL model proves that the error produced using the advanced method is noteworthy smaller than the adapted and basic methods. This verdict is in line with what we have proposed, i.e. the RMSE: advanced $\leq$ adapted $\leq$ basic. This outcome signifies the superiority of the MGL model against the MFBKM model that utilises the concept of fully-implied model in generating the option-implied volatility. RMSE of the MGL models decrease as rebalancing becomes frequent from fortnightly to daily. Table 1 provides the summary of error analysis of option-implied volatility priced using both MGL models.

The t-test and pair-wise test statistics are carried out. The option-implied information is obtained by utilising three integral approximation approaches, i.e. the Basic (trapezoidal-rule) approach, the Adapted (single-combined) approach and the Advanced (single-combined, cubic-spline) approach. The three approaches – Basic, Adapted and Advanced – are then manipulated across the different rebalancing frequency. Values of the adjusted volatility and variance implied by the MGL models are observed to be significantly different based on the t-test, across all estimation approaches. As a matter of fact, the results are consistent across the different rebalancing frequency basis.

Table 1: Summary of RMSE analysis of volatility implied from the MFIL compared to those generated from the GLI and MFBKM models across different rebalancing basis frequency.

Rebalancing Frequency		**Daily**		**Weekly**		**Fortnightly**	
Model	**Approach**	**RMSE (pts.)**	**S.D.**	**RMSE (pts.)**	**S.D.**	**RMSE (pts.)**	**S.D.**
L85	Basic	0.1936	0.0825	0.1976	0.0839	0.2000	0.0841
	Adapted	0.1908	0.0820	0.1957	0.0834	0.1973	0.0837
	Advanced	0.1697	0.0671	0.1751	0.0690	0.1769	0.0694
LCASH	Basic	0.1784	0.0795	0.1845	0.0807	0.1864	0.0809
	Adapted	0.1756	0.0790	0.1818	0.0802	0.1837	0.0804
	Advanced	0.1513	0.0640	0.1581	0.0657	0.1606	0.0661
LSTOCK	Basic	0.1875	0.0814	0.1944	0.0826	0.1967	0.0829
	Adapted	0.1846	0.0808	0.1916	0.0821	0.1939	0.0824
	Advanced	0.1629	0.0658	0.1707	0.0677	0.1737	0.0681
GLI	Basic	0.2160	0.0894	0.2047	0.0872	0.2012	0.0866
	Adapted	0.2127	0.0884	0.2013	0.0862	0.1978	0.0856
	Advanced	0.2016	0.0853	0.1900	0.0831	0.1866	0.0824
MFBKM	Basic	0.2301	0.0920	0.2301	0.0920	0.2301	0.0920
	Adapted	0.2275	0.0913	0.2275	0.0913	0.2275	0.0913
	Advanced	0.2188	0.0890	0.2188	0.0890	0.2188	0.0890

Tables 2 and 3 report the pairwise pricing comparison in terms of RMSE values percentage difference between any two option pricing models. When the MGL models are compared with the MFBKM model, all MGL models have superior pricing performance compared to the MFBKM model. This is depicted by the positive value of the pairwise percentage difference. In particular, the pairwise percentage difference between the MGL model and the MFBKM model is determined by the percentage relative difference between those two models, positive value signifies that the MGL model performs better than the MFBKM model. All MGL models are shown to be statistically different compared to the MFBKM model over all approaches and across all rebalancing frequency basis. In point of fact, the outperformance of the MGL models with weekly and fortnightly rebalancing, are moderately smaller, but are statistically significant at 1 per cent levels. The pairwise percentage differences are observed to increase as the models are estimated from basic approach to advanced approach. This implies the superiority and reliability of the advanced approach as an estimation tool in this study.

However, the MFIL models are identified to exceed the outperformance of the MFBKM model better than the GLI model. The pairwise percentage differences pointed on the GLI model result in positive values. In daily rebalancing, the models performance is statistically significantly better than the GLI model. When the rebalancing is performed weekly, both MFIL All-Cash and MFIL All-Stock models show to have statistically significant pricing performance. Only in the case of Leland (1985) model, the model outperformance is not statistically significant, except when the estimation is carried out using the basic approach. In contrast, only MFIL All-Cash model is statistically outperforms the GLI model in fortnightly rebalancing. MFIL All-Stock only appears to be statistically outranks the GLI model when the option-implied volatility is estimated using the advanced approach.

Table 2: Summary of pairwise percentage differences of mean pricing errors (RMSE) between option pricing models.

Approach	Model	L85	LCASH	LSTOCK	GLI	MFBKM
			PANEL A: Daily Rebalancing			
Basic Approach	L85	-	-7.8400**	-3.1309	11.5895**	18.8425**
	LCASH	-	-	5.1096**	21.0824**	28.9524**
	LSTOCK	-	-	-	15.1962**	22.6837**
	GLI	-	-	-	-	6.4997**
	MFBKM	-	-	-	-	-
Adapted Approach	L85	-	-7.9651**	-3.2621	11.4812**	19.2347**
	LCASH	-	-	5.1100**	21.1293**	29.5537**
	LSTOCK	-	-	-	15.2405**	23.2554**
	GLI	-	-	-	-	6.9549**
	MFBKM	-	-	-	-	-
Advanced Approach	L85	-	-10.8759**	-4.0399	18.7651**	28.8776**
	LCASH	-	-	7.6702**	33.2582**	44.6047**
	LSTOCK	-	-	-	23.7652**	34.3034**
	GLI	-	-	-	-	8.5147**
	MFBKM	-	-	-	-	-
			PANEL B: Weekly Rebalancing			
Basic Approach	L85	-	-6.6271**	-1.6159	3.5735*	16.4249**
	LCASH	-	-	5.3668**	10.9246**	24.6880**
	LSTOCK	-	-	-	5.2747**	18.3371**
	GLI	-	-	-	-	12.4079**
	MFBKM	-	-	-	-	-
Adapted Approach	L85	-	-7.1192**	-2.1340	2.8326	16.2456**
	LCASH	-	-	5.3673**	10.7146**	25.1557**
	LSTOCK	-	-	-	5.0749**	18.7803**
	GLI	-	-	-	-	13.0435**
	MFBKM	-	-	-	-	-
Advanced Approach	L85	-	-9.7209**	-2.5285	8.5124	24.9115**
	LCASH	-	-	7.9668**	20.1965**	38.3614**
	LSTOCK	-	-	-	11.3273**	28.1518**
	GLI	-	-	-	-	15.1126**
	MFBKM	-	-	-	-	-

Table 3: *Continued.*

Approach	Model	L85	LCASH	LSTOCK	GLI	MFBKM
			PANEL C: Fortnightly Rebalancing			
Basic Approach	L85	-	-6.7751**	-1.6139	0.6264	15.0551**
	LCASH	-	-	5.5363**	7.9394**	23.4167**
	LSTOCK	-	-	-	2.2770	16.9424**
	GLI	-	-	-	-	14.3389**
	MFBKM	-	-	-	-	-
Adapted Approach	L85	-	-6.8734**	-1.7338	0.2813	15.3428**
	LCASH	-	-	5.5189**	7.6827**	23.8559**
	LSTOCK	-	-	-	2.0506	17.3779**
	GLI	-	-	-	-	15.0193**
	MFBKM	-	-	-	-	-
Advanced Approach	L85	-	-9.2212**	-1.8316	5.4785	23.6388**
	LCASH	-	-	8.1401*	16.1929**	36.1978**
	LSTOCK	-	-	-	7.4466*	25.9457**
	GLI	-	-	-	-	17.2170**
	MFBKM	-	-	-	-	-

5. Conclusions

This research extends the Leland models by developing MGL models based on the implied adjusted volatility introduced in Leland models. In order to assess the models' relative performance, this study considers the different Leland models, namely the Leland (1985), Leland All-Cash and Leland All-Stock models. Using the three Leland models, this study attempts to integrate the models into model-free framework, developed by Bakshi et al. (2003) to reduce the model misspecification error introduced by the original models. This research investigates the performance of MGL models in comparison with the benchmark model, the MFBKM.

This study considers the manipulation factors, i.e. estimation approach, type of true values, as well as the rebalancing frequency basis, in assessing the option pricing performance of the models. The relative options pricing performance are compared against the benchmark option moments implied by the MFBKM model. Based on our investigation, the advanced approach delivers significantly superior estimation results relative to the other approaches considered. This is supported by the t-test carried out against the other approaches considered in estimating the option-implied (adjusted) moments. This implies the superiority and reliability of the advanced approach used in this study.

The option pricing performance of the option adjusted moments implied on the DJIA index options were assessed using RMSE. Pricing error was measured based on how the theoretical option-implied adjusted moments are priced using each model against the market-observed return data. There is a consistent consent that can be pointed out across all three approaches considered in this study, across all basis of rebalancing frequency. The MGL models significantly outperformed option-implied adjusted moments compared to the MFBKM model based on the RMSE values. This finding is further supported by the statistically significant positive results in pairwise percentage difference. We verified the option pricing abilities of the models estimated based on advanced approach to be statistically significantly superior to that of the adapted approach. This verdict is in line with what we have proposed, i.e. the RMSE: advanced $\leq$ adapted $\leq$ basic.

Above all, this outcome signifies the superiority of the MGL model that utilises the concept of fully-implied model in generating the option-implied adjusted volatility against the MFBKM model. RMSE of the MGL models decrease as rebalancing becomes frequent from fortnightly to daily. The empirical findings justify the superiority of the MGL models with daily rebalancing in pricing options compared to that of the model-free model. As our attention is drawn in investigating the pricing performance among the MGL models per se, this study discovers that MFIL models outperformed GLI models. The findings are significant across daily and weekly rebalancing frequency basis, especially when the option-implied volatility is estimated using advanced approach.

Overall, this study has developed MGL models, which integrate the conventional parametric Leland option pricing models and the nonparametric model-free framework. The study on the hybrid model made of both parametric and nonparametric is thin on the ground to our best knowledge. Thus, this study has successfully fulfilled this void. The option-implied adjusted volatility generated from the MGL models are found to significantly outperform the model-free model in option pricing accuracy in robust a manner. This could shed some light on the future refinement of the hybrid option pricing model.

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